Log Models

Log Functions Day 7:
- (2.14) Logarithmic Models

Homework: Logarithmic Models + FRQ 2 Practice

The graph and table below give data on the salary of clergy members across the US and the size of their congregations.

Congregation size	75	120	250	300	450	650	900	1500	2500	5000
Salary	42	58	70	79	88	95	110	125	138	175

What type of regression should we perform to model this data?

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Homework: Logarithmic Modelling + Calculator Practice

Log Models

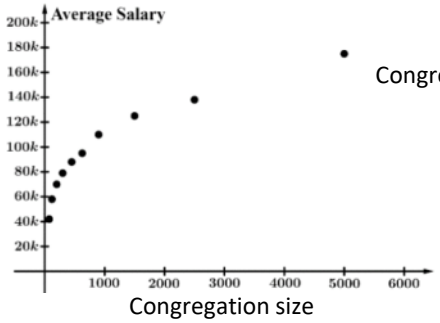
Log Models are of the form $f(x) = a + b \ln x$ or $a + b \log x$

(why does the log base not really matter?)

We use log models when input seems to change proportionally over equal-length output-value intervals

(stolen from Bryan Passwater)

The graph and table below give data on the salary of clergy members across the US and the size of their congregations.



Congregation size	75	120	250	300	450	650	900	1500	2500	5000
Salary	42	58	70	79	88	95	110	125	138	175

(stolen from Bryan Passwater)

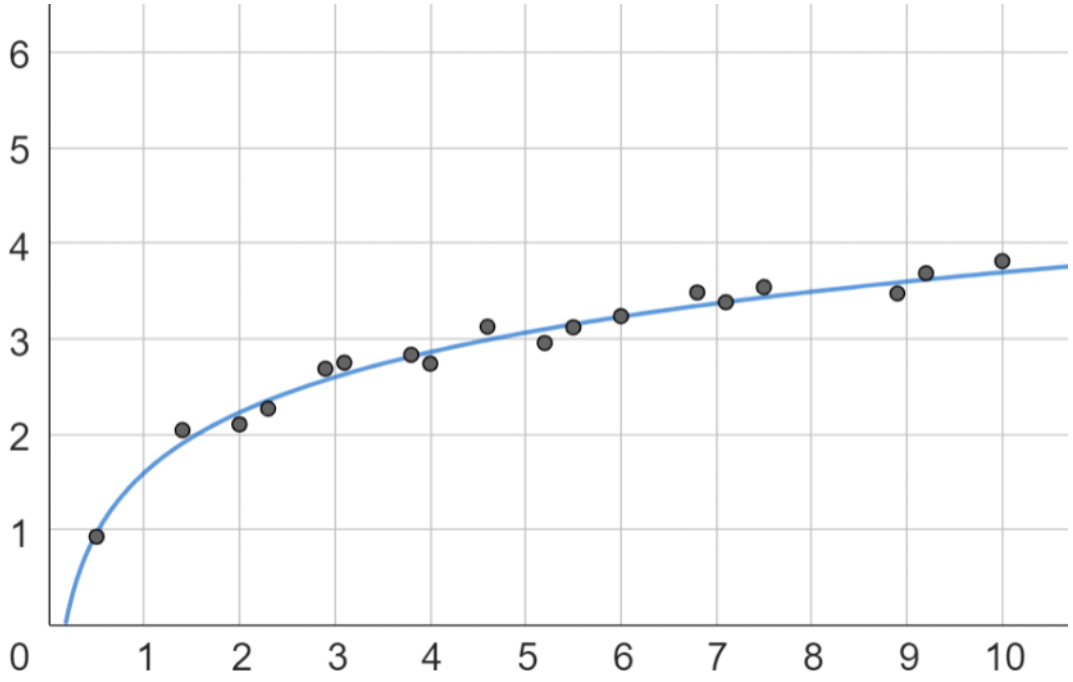
Church Size	75	120	250	300	450	650	900	1500	2500	5000
Avg. Salary (in 1000's of dollars)	42	58	70	79	88	95	110	125	138	175

A) Use the regression capabilities of your calculator to find a logarithmic function that models the data.

B) Using the model, predict the annual salary, in thousands of dollars, for a clergy member whose church size is 500 people.

C) If a clergy member makes a $\$100{,}000$ annual salary, predict the size of their congregation based on this model.

18 data points are plotted below. They are modeled well by a function of the form $f(x) = a + \log_b x$. What base logarithm would be most appropriate?



(A) $b = 2$
(B) $b = 3$
(C) $b = 4$
(D) $b = 5$

The most common method to describe sound intensity level is in decibels (dB). The intensity level β (in decibels) of a sound having intensity I (in watts per meter squared) is modeled by a function of the following form.

$$\beta(I) = a + b\log(I)$$

Noisy traffic has intensity 1×10^{-5} watts per meter squared and that corresponds with an intensity level of 70 dB.

A loud concert has intensity 1 watt per meter squared and that corresponds with an intensity level of 120 dB.

A) Using a calculator/Desmos/your impressive brains, find the value of a and b

B) The human eardrum can burst if exposed to 1×10^{4} watts per meter squared. According to the model found above, what is the predicted number of decibels that would burst the human eardrum?

The function f is of the form $f(x) = a + b\log(x-1)$

$f(6) = 7$ and $f(9) = 11$

A) Find a and b

B) Find $f(10)$

C) Find $f^{-1}(12)$

The function f is of the form $f(x) = a + b\log(x-1)$

$f(6) = 7$ and $f(9) = 11$

D) Find the average rate of change of f on the interval $6 \leq x \leq 9$

E) Use the average rate of change from part (D) to estimate $f(11)$

F) Using the shape of the graph of $f(x)$ explain how one knows that the answer to (E) overestimates the actual value of $f(11)$.

Without looking at your notes to figure out the wording, try to justify why you believe the values in the following table would be modelled well by a logarithmic function.

x	100	50	25	12.5	6.25	3.125
$g(x)$	-4	0	4	8	12	16

As a reminder, the structure of FRQ2 is:

- Context that includes two to four points.
- The type of equation we'd like to use to model the context.

- A:
  - Part i: Set up a system of equations to solve for a, b, c (no work necessary)
  - Part ii: Solve for a, b, c (no work necessary)
- B:
  - Find an average rate of change
  - Apply the average rate of change (often, use average rate of change to estimate)
  - Reason using the average rate of change (often, use concavity to explain why estimate based on AROC is greater/less than value from model)
- C: Explain limitation of the model (domain restriction, range restrictions, other)

Part A

Part i: No work necessary

Part ii: No work necessary

Part B

- The work for average rate of change should look like a subtraction problem divided by a subtraction problem
- Explaining why estimates based on average rate of change over/under estimate values of a model should include:
  - Quick sketch with t-values and graphs labelled
  - Talk about concavity of the model
  - Refer to the estimates based on AROC as outputs of a secant line
  - t-values of secant line
  - t-values the problem is about
  - Which graph is on top

Part C

For domain/range restrictions, write something like:

For _____ input/output values,
the model predicts _____,
but in reality _____.
So we restrict _____.